MP201 – Vector Calculus & Fourier Analysis

Problem Set 1

Due by 5pm on Friday, 29 September 2017

(Please write your name and tutorial day on the front of your assignment.)

1. Given the two functions

$$f(x,y) = y^3 - 2\cos(xy), \qquad g(x,y) = e^{-3x}\left(y - y^2\right),$$

compute the following partial derivatives:

$$\text{(a) } \frac{\partial f}{\partial x}, \text{ (b) } \frac{\partial g}{\partial y}, \text{ (c) } \frac{\partial}{\partial x}\left(fg\right).$$

2. Consider the function

$$G(x,y,z) \quad = \quad \frac{1}{\sqrt{x^2+y^2+z^2}}.$$

   (a) Compute the following partial derivatives:

$$\text{(i) } \frac{\partial G}{\partial z}, \text{ (ii) } \frac{\partial^2 G}{\partial y^2}, \text{ (iii) } \frac{\partial^2 G}{\partial x \partial y}.$$

   (b) Show that

$$\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} + \frac{\partial^2 G}{\partial z^2} \quad = \quad 0.$$

3. Let (x,y,z) be the usual Cartesian coordinates in three dimensions, and let $a(x,y)$, $b(x,z)$ and $c(y,z)$ be three functions that depend only the variables indicated. If $f(x,y,z)$ is the sum of these three functions, show that

$$\frac{\partial^3 f}{\partial x \partial y \partial z} \quad = \quad 0.$$

4. Any equation which relates the partial derivatives of a function to each other is called a *partial differential equation* (PDE). One of the most famous is the heat equation, which says that if $T(t,x)$ is the temperature at time t at a distance x along a thin metal rod, then the function T must satisfy the PDE

$$\frac{\partial T}{\partial t} \quad = \quad \alpha \frac{\partial^2 T}{\partial x^2}$$

where α is the metal's thermal diffusivity (a physical property of the metal in question). Show that the function

$$T(t,x) \quad = \quad e^{-4t}\cos\left(\frac{2x}{\sqrt{\alpha}}\right) + 2e^{-t}\sin\left(\frac{x}{\sqrt{\alpha}}\right) - 5x$$

satisfies this PDE.