# Solution to EE112 Exam, Spring 2017-18



P.1

(a)

(i) We know that if $F(s)$ is the LT of $f(t)$, then the LT of $e^{at}f(t)$ is $F(s-a)$; $t$ has LT $1/s^2$ and $t^3$ has LT $\frac{3!}{s^4} = 6/s^4$, so,

$$e^{-t}t \xrightarrow{LT} \frac{1}{(s-(-1))^2} = \frac{1}{(s+1)^2}$$

$$e^{-t}t^3 \xrightarrow{LT} \frac{6}{(s-(-1))^4} = \frac{6}{(s+1)^4}$$

and thus

$$\boxed{L[e^{-t}(t+t^3)] = \frac{1}{(s+1)^2} + \frac{6}{(s+1)^4}.}$$

[4 marks]

(ii) Due to the linearity of $LT^{-1}$, we can rewrite this as

$$LT^{-1}\left[\frac{3-5s}{s^2+4}\right] = 3\,LT^{-1}\left[\frac{1}{s^2+4}\right] - 5\,LT^{-1}\left[\frac{s}{s^2+4}\right]$$

$$= \frac{3}{2}LT^{-1}\left[\frac{2}{s^2+(2)^2}\right] - 5\,LT^{-1}\left[\frac{s}{s^2+(2)^2}\right]$$

Since the LT of $\cos(\omega t)$ is $\frac{s}{s^2+\omega^2}$ and the LT of $\sin(\omega t)$ is $\frac{\omega}{s^2+\omega^2}$, we see the above $LT^{-1}$, are $\sin(2t)$ and $\cos(2t)$ respectively, so

$$\boxed{LT^{-1}\left[\frac{3-5s}{s^2+4}\right] = \frac{3}{2}\sin(2t) - 5\cos(2t)}$$

[4 marks]

(b) $\vec{a} = \hat{i} + \hat{k}$, $\vec{b} = -2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{c} = 4\hat{i} - 2\hat{k}$ here, so:

(i) $\vec{a}\cdot\vec{c} = (\hat{i}+\hat{k})\cdot(4\hat{i}-2\hat{k}) = (1)(4) + (0)(0) + (1)(-2) = \boxed{2}$ [3 marks]

(ii)

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ -2 & 1 & 3 \end{vmatrix} = \hat{i}\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix}$$

$$= \boxed{-\hat{i} - 5\hat{j} + \hat{k}}$$

[3 marks]

(iii) $\hat{j}\times\vec{b} = \hat{j}\times(-2\hat{i}+\hat{j}+3\hat{k}) = -2(-\hat{k}) + \vec{0} + 3(\hat{i}) = 3\hat{i} + 2\hat{k}$

$\vec{a}\times\hat{k} = (\hat{i}+\hat{k})\times\hat{k} = -\hat{j}$

and so

$(\hat{j}\times\vec{b})\cdot(\vec{a}\times\hat{k}) = (3\hat{i}+3\hat{k})\cdot(-\hat{j}) = \boxed{0}$ [3 marks]

(iv) $\vec{c}\cdot\vec{a} = \vec{a}\cdot\vec{c} = 2$ from (i), so

$2(\vec{c}\cdot\vec{a})\hat{j} + 3\vec{a} = 2(2)\hat{j} + 3(\hat{i}+\hat{k}) = \boxed{3\hat{i} + 4\hat{j} + 3\hat{k}}$ [3 marks]

(c) The line has $x(t) = t+1$, $y(t) = 2t+1$ and $z(t) = t+1$, so for the point which is shared by both the line and the plane, $t$ must be such that $2x(t) - 5y(t) + z(t) = 5$, or

$$2(t+1) - 5(2t+1) + (t+1) = -7t - 2 = 5$$

Thus, $-7t = 7$ or $t = -1$. Putting this into $x(t)$, $y(t)$ and $z(t)$ gives $x(-1) = z(-1) = 0$, $y(-1) = -1$, so $\boxed{\vec{r}(-1) = -\hat{j}}$ or $\boxed{(0,-1,0)}$ (either is fine) is the point of intersection. [3 marks]

(d) To find $\kappa$ and $\hat{N}$, we need $\vec{u}(t)$ and $\hat{u}(t)$. $\vec{u}(t) = \frac{d\vec{r}}{dt}(t)$, of course, which gives

$$\vec{u}(t) = 4t\,\hat{i} - 4t\cos(t^2)\,\hat{j} - 4t\sin(t^2)\,\hat{k}.$$

Thus,

$$|\vec{u}| = \sqrt{(4t)^2 + (-4t\cos(t^2))^2 + (-4t\sin(t^2))^2}$$
$$= \sqrt{16t^2 + 16t^2\cos^2(t^2) + 16t^2\sin^2(t^2)}$$
$$= \sqrt{32t^2} = 4\sqrt{2}\,t \quad (\text{since } t>0).$$

Also

$$\hat{u}(t) = \frac{\vec{u}(t)}{|\vec{u}(t)|} = \frac{4t\,\hat{i} - 4t\cos(t^2)\,\hat{j} - 4t\sin(t^2)\,\hat{k}}{4\sqrt{2}\,t}$$

$$= \frac{1}{\sqrt{2}}\left[\hat{i} - \cos(t^2)\,\hat{j} - \sin(t^2)\,\hat{k}\right]$$

$$\frac{d\hat{u}}{dt} = \frac{1}{\sqrt{2}}\left(2t\sin(t^2)\,\hat{j} - 2t\cos(t^2)\,\hat{k}\right) = \sqrt{2}\,t\left[\sin(t^2)\,\hat{j} - \cos(t^2)\,\hat{k}\right],$$

and this has magnitude

$$\left|\frac{d\hat{u}}{dt}\right| = \sqrt{(\sqrt{2}t)^2\left[(\sin(t^2))^2 + (-\cos(t^2))^2\right]} = \sqrt{2}\,t.$$

Thus, we immediately get the curvature

$$\kappa = \frac{\left|\frac{d\hat{u}}{dt}\right|}{|\vec{u}|} = \frac{\sqrt{2}\,t}{4\sqrt{2}\,t} = \boxed{1/4}$$ [3 marks]

and the principal unit normal vector

$$\hat{N} = \frac{\frac{d\hat{u}}{dt}}{\left|\frac{d\hat{u}}{dt}\right|} = \frac{\sqrt{2}\,t\left[\sin(t^2)\,\hat{j} - \cos(t^2)\,\hat{k}\right]}{\sqrt{2}\,t} = \boxed{\sin(t^2)\,\hat{j} - \cos(t^2)\,\hat{k}}$$ [3 marks]

(e) $A = \begin{pmatrix} 12 & 0 \\ 0 & -7 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$, so:

(i) $\boxed{A^T = \begin{pmatrix} 12 & 0 & 5 \\ 0 & -7 & 3 \end{pmatrix}}$ (1st row → 1st column, 2nd row → 2nd column, 3rd row → 3rd column) [2 marks]

(ii) $\boxed{B^T = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}}$ [2 marks]

(Since $B$ is square, just "reflect" across its diagonal.)

(iii) We have $A^T$ fr (i), so

$$B(A^T) = \begin{pmatrix}2&0\\-1&1\end{pmatrix}\begin{pmatrix}12&0&5\\0&-7&3\end{pmatrix} = \begin{pmatrix}(2)(12)+(0)(0) & (2)(0)+(0)(-7) & (2)(5)+(0)(3)\\ (-1)(12)+(1)(0) & (-1)(0)+(1)(-7) & (-1)(5)+(1)(3)\end{pmatrix}$$

$$= \boxed{\begin{pmatrix}24&0&10\\-12&-7&-2\end{pmatrix}}$$

[2 marks]

(iv) $AB$ is

$$AB = \begin{pmatrix}12&0\\0&-7\\5&3\end{pmatrix}\begin{pmatrix}2&0\\-1&1\end{pmatrix} = \begin{pmatrix}24&0\\7&-7\\7&3\end{pmatrix},$$

so

$$\boxed{(AB)^T = \begin{pmatrix}24&7&7\\0&-7&3\end{pmatrix}.}$$

[2 marks]

(f) This is an upper triangular matrix, so its determinant is just the product of its diagonal entries, i.e.

$$\det\begin{pmatrix}5&-13&10\\0&-2&1\\0&0&3\end{pmatrix} = (5)(-2)(3) = \boxed{-30}.$$

[3 marks]

The trace is the sum of the diagonal entries, i.e.

$$\mathrm{tr}\begin{pmatrix}5&-13&10\\0&-2&1\\0&0&3\end{pmatrix} = (5)+(-2)+(3) = \boxed{6}$$

[3 marks]

(g) First we write the two eq'ns as a single matrix eq'n:

$$\begin{pmatrix}2&1\\-9&3\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}-2\\4\end{pmatrix}.$$

So we create the augmented matrix, and use row operations to reduce:

$$\left(\begin{array}{cc|c}2&1&-2\\-9&3&4\end{array}\right) \xrightarrow[+(2^{nd}\text{ row})]{\frac{9}{2}\times(1^{st}\text{ row})} \left(\begin{array}{cc|c}2&1&-2\\0&15/2&-5\end{array}\right)$$

So our two eq'ns are now

$$2x_1+x_2 = -2, \quad 15/2\, x_2 = -5.$$

The last immediately gives $x_2 = -2/3$. Putting this into the first yields

$$2x_1 - 2/3 = -2 \Rightarrow 2x_1 = -4/3$$

and so $\boxed{x_1 = x_2 = -2/3}$ is the sol'n.

[7 marks]

P.2

(a) First we need to LT the ODE using the rule $\frac{dy}{dt}(t) \xrightarrow{LT} sY(s) - y(0)$, where $Y(s)$ is the LT of $y(t)$. We also know $te^t \xrightarrow{LT} \frac{1}{(s-1)^2}$, so
$\frac{dy}{dt} + 3y = -8te^t \xrightarrow{LT} sY(s) - y(0) + 3Y(s) = \frac{-8}{(s-1)^2}$. Here, $y(0) = 0$,
so we get $(s+3)Y(s) = -\frac{8}{(s-1)^2}$, or

$$Y(s) = -\frac{8}{(s-1)^2(s+3)}$$

so if we knew the function with $-\frac{8}{(s-1)^2(s+3)}$ as its LT, we'd be done. To determine this, we could use some of the identities given in the exam, but let's use the partial fraction method instead: since the denominator of the transfer function is $(s-1)^2(s+3)$, we know there must exist constants $A$, $B$ and $C$ such that

$$-\frac{8}{(s-1)^2(s+3)} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+3}$$

Multiplying both sides by $(s-1)^2(s+3)$ gives

$$-8 = A(s-1)(s+3) + B(s+3) + C(s-1)^2$$
$$= (A+C)s^2 + (2A+B-2C)s + (-3A+3B+C)$$

If this holds for all values of $s$, the coefficients of each power of $s$ must be the same on both sides, i.e.

$$A+C = 0, \quad 2A+B-2C = 0, \quad -3A+3B+C = -8$$

which can be solved to give $A = \frac{1}{2}$, $B = -2$, $C = -\frac{1}{2}$. Thus,

$$Y(s) = \frac{1}{2(s-1)} - \frac{2}{(s-1)^2} - \frac{1}{2(s+3)}$$

Since $\frac{1}{(s-a)^n}$ is the LT of $\frac{1}{(n-1)!}t^{n-1}e^{at}$, we see $\frac{1}{s-1} \xrightarrow{LT^{-1}} e^t$, $\frac{1}{(s-1)^2} \xrightarrow{LT^{-1}} te^t$ and $\frac{1}{s+3} \xrightarrow{LT^{-1}} e^{-3t}$, so

$$Y(s) = \frac{1}{2(s-1)} - \frac{2}{(s-1)^2} - \frac{1}{2(s+3)} \xrightarrow{LT^{-1}} \boxed{y(t) = \frac{1}{2}e^t - 2te^t - \frac{1}{2}e^{-3t}}$$ [10 marks]

is the soln.

(b) To find the eigenvalues, we compute $\det(A - \lambda I)$ and find the roots of the resulting characteristic polynomial. For the matrix given

$$\det(A-\lambda I) = \begin{vmatrix} 9-\lambda & 0 & 0 \\ 0 & -3-\lambda & 1 \\ 0 & 6 & 2-\lambda \end{vmatrix} = (9-\lambda)\begin{vmatrix} -3-\lambda & 1 \\ 6 & 2-\lambda \end{vmatrix} = (9-\lambda)\left[(-3-\lambda)(2-\lambda) - 6\right]$$

$$= -(\lambda-9)[\lambda^2+\lambda-12] = -(\lambda-9)(\lambda-3)(\lambda+4)$$

and so we see the eigenvalues are $\boxed{\lambda_1 = -4,\ \lambda_2 = 3 \text{ and } \lambda_3 = 9}$. [2 marks each]

To find an eigenvector, we pick one of the eigenvalues $\lambda_i$ and try to find a column vector $\vec{K}_i$ satisfying $(A-\lambda_i I)\cdot\vec{K}_i = 0$. There will always be an ambiguity in any such eigenvector, since it's only defined up to an arbitrary multiplicative constant, so we just pick one possible one.

$\lambda_1 = -4$: here, $\vec{K}_1 = \begin{pmatrix}\alpha_1\\ \beta_1\\ \gamma_1\end{pmatrix}$ such that

$$\left[\begin{pmatrix}9&0&0\\0&-3&1\\0&6&2\end{pmatrix} - (-4)\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right]\begin{pmatrix}\alpha_1\\ \beta_1\\ \gamma_1\end{pmatrix} = \begin{pmatrix}13&0&0\\0&1&1\\0&6&6\end{pmatrix}\begin{pmatrix}\alpha_1\\ \beta_1\\ \gamma_1\end{pmatrix} = \begin{pmatrix}13\alpha_1\\ \beta_1+\gamma_1\\ 6\beta_1+6\gamma_1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

we see immediately that $\alpha_1 = 0$, and the other two give $\beta_1+\gamma_1 = 0$ and $6\beta_1+6\gamma_1 = 0$. These are not independent, so we only have a single eq'n for $\beta_1+\gamma_1 = 0$; thus, we pick any two (nonzero) numbers which satisfy this. I'll pick $\beta_1 = 1$ and $\gamma_1 = -1$, so an eigenvector for $\lambda_1 = -4$ is $\boxed{\vec{K}_1 = \begin{pmatrix}0\\1\\-1\end{pmatrix}}$ [3 marks]

$\lambda_2 = 3$: $\vec{K}_2 = \begin{pmatrix}\alpha_2\\ \beta_2\\ \gamma_2\end{pmatrix}$ gives

$$\left[\begin{pmatrix}9&0&0\\0&-3&1\\0&6&2\end{pmatrix} - (3)\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right]\begin{pmatrix}\alpha_2\\ \beta_2\\ \gamma_2\end{pmatrix} = \begin{pmatrix}6&0&0\\0&-6&1\\0&6&-1\end{pmatrix}\begin{pmatrix}\alpha_2\\ \beta_2\\ \gamma_2\end{pmatrix} = \begin{pmatrix}6\alpha_2\\ -6\beta_2+\gamma_2\\ 6\beta_2-\gamma_2\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

so $\alpha_2 = 0$ and $-6\beta_2+\gamma_2 = 0$. ($6\beta_2-\gamma_2$ ~~= 0 is redundant~~) Thus, if we pick $\beta_2 = 1$ and $\gamma_2 = 6$, we get the eigenvector $\boxed{\vec{K}_2 = \begin{pmatrix}0\\1\\6\end{pmatrix}}$ [3 marks]

$\lambda_3 = 9$: $\vec{K}_3 = \begin{pmatrix}\alpha_3\\ \beta_3\\ \gamma_3\end{pmatrix}$ gives

$$\left[\begin{pmatrix}9&0&0\\0&-3&1\\0&6&2\end{pmatrix} - (9)\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right]\begin{pmatrix}\alpha_3\\ \beta_3\\ \gamma_3\end{pmatrix} = \begin{pmatrix}0&0&0\\0&-12&1\\0&6&-7\end{pmatrix}\begin{pmatrix}\alpha_3\\ \beta_3\\ \gamma_3\end{pmatrix} = \begin{pmatrix}0\\ -12\beta_3+\gamma_3\\ 6\beta_3-7\gamma_3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

So $-12\beta_3+\gamma_3 = 0$ and $6\beta_3-7\gamma_3 = 0$. The first gives $\gamma_3 = 12\beta_3$, so the second becomes $6\beta_3 - 7(12\beta_3) = (6-84)\beta_3 = -78\beta_3 = 0$, so $\beta_3 = 0$, and thus $\gamma_3 = 0$. But $\alpha_3$ can be whatever we like (except zero), so let's take $\alpha_3 = 1$ to obtain $\boxed{\vec{K}_3 = \begin{pmatrix}1\\0\\0\end{pmatrix}}$ as the third eigenvector. [3 marks]

P. 3

(a) We can write these two planes as $\vec{n}_1\cdot\vec{r} = D_1$ and $\vec{n}_2\cdot\vec{r} = D_2$, with $\vec{n}_1 = \hat{i}+\hat{j}-2\hat{k}$, $D_1 = 4$, $\vec{n}_2 = \hat{i}-\hat{j}+\hat{k}$, $D_2 = -2$. The direction vector of the line of intersection between the two planes is $\vec{a} = \vec{n}_1 \times \vec{n}_2$, or

$$\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & -2 \\ -1 & 2 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= \hat{i}(2-2) - \hat{j}(2-(-2)) + \hat{k}(-1-1)$$

$$= -4\hat{j} - 2\hat{k}$$

That's the direction vector; we still need a point on the line of intersection, i.e. $x_1, y_1$ and $z_1$ that satisfy both $x_1+y_1-2z_1=4$ and $x_1-y_1+2z_1=-2$. Let's see if we can find such a point in the $xy$ plane, i.e. $z_1=0$. If this point exists, it satisfies both $x_1+y_1=4$ and $x_1-y_1=-2$. Adding these eq's together gives $2x_1=2$, or $x_1=1$, and $x_1+y_1=4 \Rightarrow y_1=3$. Then, $(x_1,y_1,z_1)=(1,3,0)$ is a point in both planes, i.e. on the line of intersection. So now that we have a direction vector $\vec{a}=-4\hat{j}-2\hat{k}$ and a point on the line with position vector $\vec{r}_1=\hat{i}+3\hat{j}$, then we know that the line can be expressed as $\vec{r}(t)=\vec{r}_1+t\vec{a}$ for $t\in\mathbb{R}$, i.e.

$$\boxed{\vec{r}(t) = \hat{i} + (3-4t)\hat{j} - 2t\hat{k}}$$

(b) I'm going to use the ~~adjugate~~ adjugate matrix method, since it's the one I'm most familiar with. First, we need the nine cofactors of the matrix, given by

$$C_{ij} = (-1)^{i+j}\begin{vmatrix} i\text{th column} \\ \& \\ j\text{th row} \\ \text{deleted} \end{vmatrix}$$

so

$$C_{11} = (-1)^{1+1}\begin{vmatrix} -2 & -1 \\ -4 & 3 \end{vmatrix} = -10,\ C_{22} = (-1)^{1+2}\begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix} = -8,\ C_{33} = (-1)^{1+3}\begin{vmatrix} 2 & 2 \\ 2 & -4 \end{vmatrix} = -4$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 1 \\ -4 & 3 \end{vmatrix} = 2,\ C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1,\ C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -2 \\ 2 & -4 \end{vmatrix} = 0,$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 1 \\ -2 & -1 \end{vmatrix} = 4,\ C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = 3,\ C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ 2 & -2 \end{vmatrix} = 2$$

so the cofactor matrix is

$$C = \begin{pmatrix} -10 & -8 & -4 \\ 2 & 1 & 0 \\ 4 & 3 & 2 \end{pmatrix}.$$

We also need the determinant: expanding along the top row gives

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & -2 & -1 \\ 2 & -4 & 3 \end{vmatrix} = (1)\begin{vmatrix} -2 & -1 \\ -4 & 3 \end{vmatrix} - (-2)\begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix} + (1)\begin{vmatrix} 2 & -2 \\ 2 & -4 \end{vmatrix}$$

$$= -10 + 16 - 4 = 2,$$

This means the inverse is

⑦

$$\begin{pmatrix} 1 & -2 & 1 \\ 2 & 2 & -1 \\ 2 & -4 & 3 \end{pmatrix}^{-1} = \frac{1}{2} C^T = \frac{1}{2}\begin{pmatrix} -10 & 2 & 4 \\ -8 & 1 & 3 \\ -4 & 0 & 2 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} -5 & 1 & 2 \\ -4 & 1/2 & 3/2 \\ -2 & 0 & 1 \end{pmatrix}}$$

[15 marks]

P.4

(a) First, rewrite this as a single matrix eq'n:

$$\begin{pmatrix} -1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$$

and set up the augmented matrix, and row reduce:

$$\left(\begin{array}{ccc|c} -1 & 1 & 1 & -3 \\ 2 & 4 & 1 & 3 \\ 2 & 0 & 1 & 1 \end{array}\right) \xrightarrow[+(2^{nd}\text{ row})]{2\times(1^{st}\text{ row})} \left(\begin{array}{ccc|c} -1 & 1 & 1 & -3 \\ 0 & 6 & 3 & -3 \\ 2 & 0 & 1 & 1 \end{array}\right) \xrightarrow[+(3^{rd}\text{ row})]{2\times(1^{st}\text{ row})} \left(\begin{array}{ccc|c} -1 & 1 & 1 & -3 \\ 0 & 6 & 3 & -3 \\ 0 & 2 & 3 & -5 \end{array}\right)$$

$$\xrightarrow[+(3^{rd}\text{ row})]{(-\frac{1}{3})\times(2^{nd}\text{ row})} \left(\begin{array}{ccc|c} -1 & 1 & 1 & -3 \\ 0 & 6 & 3 & -3 \\ 0 & 0 & 2 & -4 \end{array}\right)$$

so the new eq's are $-I_1+I_2+I_3=-3$, $6I_2+3I_3=-3$ and $2I_3=-4$,
so $\boxed{I_3=-2}$. Putting this into the second gives $6I_2+3(-2)=-3$,
or $\boxed{I_2=1/2}$, and using the first,

$$I_1 = I_2+I_3+3 = \tfrac{1}{2}+(-2)+3 = \boxed{3/2}$$

[5 marks each]

(b) The tangent vector to the curve is

$$\vec{u}(t) = \frac{d\vec{r}}{dt}(t) = 6\hat{j} - 6\sinh(3t)\hat{k}$$

which has length

$$|\vec{u}(t)| = \sqrt{(0)^2+(6)^2+(-6\sinh(3t))^2} = \sqrt{36+36\sinh^2(3t)}$$

$$= \sqrt{36(1+\sinh^2(3t))}$$

Since $\cosh^2 x - \sinh^2 x = 1$ for any $x$, $1+\sinh^2(3t) = \cosh^2(3t)$, so

$$|\vec{u}(t)| = \sqrt{36\cosh^2(3t)} = 6\cosh(3t).$$

Then, the arclength of this curve from $t_1 = 0$ to $t_2 = 1$ is

$$S = \int_0^1 |\bar{u}(t)|\,dt = \int_0^1 6\cosh(3t)\,dt$$

$$= 6 \left[ \frac{1}{3} \sinh(3t) \right]_0^1 = 2\sinh(3) - 2\sinh(0)$$

$\sinh(0) = 0$, and using a calculator we find $\sinh(3) \approx 10.02$, so

$$\boxed{S = 2\sinh(3) \approx 20.04}$$

[10 marks]