Maynooth University
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OLLSCOIL NA hÉIREANN MÁ NUAD
THE NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

BE in Electronic Engineering with Communications
BE in Electronic Engineering with Computers
BE in Electronic Engineering
BE in Robotics and Intelligent Devices

YEAR 1
SEMESTER 2
2017–2018

Engineering Mathematics II
EE112

Dr. P. Watts

Time allowed: 2 hours

Answer Question 1 and any two others

Question 1 carries 50 marks and all others carry 25 marks each

1. **This Question Is Compulsory**

(a) Find the following Laplace transform and inverse Laplace transform:

$$\text{(i)} \qquad L\left[e^{-t}\left(t+t^3\right)\right],$$
$$\text{(ii)} \qquad L^{-1}\left[\frac{3-5s}{s^2+4}\right].$$

[8 marks]

(b) If $\vec{a} = \hat{\imath} + \hat{k}$, $\vec{b} = -2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{c} = 4\hat{\imath} - 2\hat{k}$, obtain the following:

$$\text{(i)} \qquad \vec{a} \cdot \vec{c},$$
$$\text{(ii)} \qquad \vec{a} \times \vec{b},$$
$$\text{(iii)} \qquad (\hat{\jmath} \times \vec{b}) \cdot (\vec{a} \times \hat{k}),$$
$$\text{(iv)} \qquad 2(\vec{c} \cdot \vec{a})\hat{\jmath} + 3\vec{a}.$$

[12 marks]

(c) Find the point at which the plane $2x - 5y + z = 5$ and the line $\vec{r}(t) = (t+1)\hat{\imath} + (2t+1)\hat{\jmath} + (t+1)\hat{k}$ (where t is a real number) intersect.

[3 marks]

(d) Compute the curvature and principal unit normal vector for the curve $\vec{r}(t) = 2t^2\hat{\imath} - 2\sin(t^2)\hat{\jmath} + 2\cos(t^2)\hat{k}$ for $t > 0$.

[6 marks]

(e) For the two matrices

$$A = \begin{pmatrix} 12 & 0 \\ 0 & -7 \\ 5 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix},$$

Find (i) A^{T}, (ii) B^{T}, (iii) $B(A^{\mathrm{T}})$ and (iv) $(AB)^{\mathrm{T}}$.

[8 marks]

(f) Find the determinant and trace of the matrix

$$\begin{pmatrix} 5 & -13 & 10 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{pmatrix}.$$

[6 marks]

(g) Solve the following system of simultaneous equations using Gauss-Jordan elimination:

$$\begin{aligned} 2x_1 + x_2 &= -2, \\ -9x_1 + 3x_2 &= 4. \end{aligned}$$

[7 marks]

2. (a) Solve the following differential equation using Laplace transforms:

$$\frac{\mathrm{d}y}{\mathrm{d}t} + 3y = -8te^{t}$$

where $y(0) = 0$.

[10 marks]

(b) Find the eigenvalues of the matrix

$$\begin{pmatrix} 9 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 6 & 2 \end{pmatrix}$$

and determine their associated eigenvectors.

[15 marks]

3. (a) Find the line of intersection, expressed in vector form, between the planes $x + y - 2z = 4$ and $x - y + 2z = -2$.

[10 marks]

(b) Using any method you like, find the inverse of the matrix

$$\begin{pmatrix} 1 & -2 & 1 \\ 2 & -2 & -1 \\ 2 & -4 & 3 \end{pmatrix}.$$

[15 marks]

4. (a) A particular circuit has three resistors such that the currents I_1, I_2 and I_3 passing through them satisfy the equations

$$\begin{aligned} -I_1 + I_2 + I_3 &= -3, \\ 2I_1 + 4I_2 + I_3 &= 3, \\ 2I_1 + I_3 &= 1. \end{aligned}$$

Find I_1, I_2 and I_3 using Gauss-Jordan elimination.

[15 marks]

(b) Consider the curve given by

$$\vec{r}(t) = 6t\hat{\jmath} - 2\cosh(3t)\hat{k},$$

for $0 \le t \le 1$. Find the total arc length of this curve.

[10 marks]

USEFUL FORMULAE

Laplace Transforms

Table of Laplace Transforms

$f(t) = L^{-1}[F(s)]$	$F(s) = L[f(t)]$
t^n	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\cos(\omega t)$	$\dfrac{s}{s^2+\omega^2}$
$\sin(\omega t)$	$\dfrac{\omega}{s^2+\omega^2}$
$\cosh(at)$	$\dfrac{s}{s^2-a^2}$
$\sinh(at)$	$\dfrac{a}{s^2-a^2}$

Laplace Transform Theorems

$$\begin{aligned}
L\left[af(t)+bg(t)\right] &= aL\left[f(t)\right]+bL\left[g(t)\right], \\
L\left[e^{at}f(t)\right] &= F(s-a), \\
L\left[f(at)\right] &= \frac{1}{a}F\left(\frac{s}{a}\right), \\
L\left[f'(t)\right] &= sF(s)-f(0), \\
L\left[f''(t)\right] &= s^2F(s)-sf(0)-f'(0), \\
L\left[\int_0^t f(\tau)\,\mathrm{d}\tau\right] &= \frac{1}{s}F(s).
\end{aligned}$$

In all of the above, $n = 0, 1, 2, \ldots$ and ω, a and b are constants.

Vectors & Curves

vector triple product: $\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \left(\vec{A}\cdot\vec{C}\right)\vec{B}-\left(\vec{A}\cdot\vec{B}\right)\vec{C}$

arc length of a curve between t_1 and t_2 : $S = \displaystyle\int_{t_1}^{t_2} \left|\vec{u}(t)\right|\,\mathrm{d}t$

curvature of a curve: $\kappa = \dfrac{\left|\frac{\mathrm{d}\hat{u}}{\mathrm{d}t}\right|}{|\vec{u}|}$

principal unit normal vector of a curve: $\hat{N} = \dfrac{\frac{\mathrm{d}\hat{u}}{\mathrm{d}t}}{\left|\frac{\mathrm{d}\hat{u}}{\mathrm{d}t}\right|}$