EE112 – Engineering Mathematics II

Problem Set 9

Due by 5pm on Friday, 20 April 2018

1. Identify the following matrices as symmetric, antisymmetric, hermitian, antihermitian, diagonal, upper-triangular, lower-triangular or none of these. (Note: list *all* that apply; for example, since the matrix

$$M = \begin{pmatrix} 1 & 0 & -3 \\ 0 & 2 & -2 \\ -3 & -2 & 0 \end{pmatrix}$$

satisfies both $M^{\mathrm{T}} = M$ and $M^{\mathrm{T}} = M^*$, you would identify it as both symmetric and hermitian.)

$$\text{(i)} \begin{pmatrix} 1 & 1-i \\ 1+i & i \end{pmatrix} \qquad \text{(ii)} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ 1 & 6i & i \end{pmatrix}$$

$$\text{(iii)} \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix} \qquad \text{(iv)} \begin{pmatrix} i & 2i \\ 2i & 2i \end{pmatrix}$$

$$\text{(v)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \text{(vi)} \begin{pmatrix} -1 & 1 & -7 \\ 1 & 4 & -5i \\ 7 & 5i & 2 \end{pmatrix}$$

2. The cyclicity property of the trace says that, for any two $m \times m$ matrices A and B, $\mathrm{tr}\,(AB) = \mathrm{tr}\,(BA)$. This does *not* mean that $\mathrm{tr}\,(AB) = (\mathrm{tr}\,(A))(\mathrm{tr}\,(B))$, as we now show:

   (a) Let A and B be the 3×3 matrices given above in Problem 1(ii) and Problem 1(vi). Compute $\mathrm{tr}\,(AB)$ and $\mathrm{tr}\,(BA)$ and show that they are equal.

   (b) Compute $\mathrm{tr}\,(A)$ and $\mathrm{tr}\,(B)$ and show that their product $(\mathrm{tr}\,(A))(\mathrm{tr}\,(B))$ is not equal to $\mathrm{tr}\,(AB)$

3. Compute the determinant of each of the matrices in Problem 1.