EE112 – Engineering Mathematics II

Problem Set 7

Due by 5pm on Friday, 6 April 2018

1. Find the tangent vector $\vec{u}(t)$ and unit tangent vector $\hat{u}(t)$ for each of the following parametrised curves:

   (a) $\vec{r}(t) = -(2+t^2)\hat{\imath} - t^2\hat{\jmath} + (6+t^2)\hat{k}$ for $0 \le t \le 1$.

   (b) $x(t) = 1-t$, $y(t) = 1+t$, $z(t) = t^2+1$ for $t \in \mathbb{R}$.

   (c) $\vec{r}(t) = te^{-2t}(\hat{\imath} - \hat{\jmath}) + 2t^3\hat{k}$ for $t \ge 0$.

2. If a curve $\mathcal{C}$ is parametrised by $\vec{r}(t)$ for $t \in [t_1, t_2]$, then the total arc length of the curve is

$$S = \int_{t_1}^{t_2} \left| \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}(t) \right| \mathrm{d}t.$$

   Suppose $\mathcal{C}$ is the lower half of a circle of radius A.

   (a) Using only what you know from elementary geometry, predict what value of S the above integral should give.

   (b) Now we parametrise the semicircle by

$$\vec{r}(t) = A\cos\left(\sqrt{t}\right)\hat{\imath} + A\sin\left(\sqrt{t}\right)\hat{\jmath}$$

   where t goes from π^2 to $4\pi^2$. Compute the arc length of this semicircle using the integral formula above and comment on how it agrees or disagrees with (a).

3. (a) Find the curvature $\kappa(t)$ and principal unit normal vector $\hat{N}(t)$ for the curve given in Problem 1(b).

   (b) Show that κ reaches its maximum value at the point $(1, 1, 1)$.