EE112 – Engineering Mathematics II

Problem Set 2

Due by 5pm on Friday, 16 February 2018

1. In this problem, we want to use the partial fraction expansion method to show that

$$L^{-1}\left[\frac{12}{s\,(s^2-8s+12)}\right] \quad = \quad 1-\frac{3}{2}\,e^{2t}+\frac{1}{2}\,e^{6t}.$$

   (a) Since $s(s^2-8s+12)=s(s-2)(s-6)$, then there must be constants A_1, A_2 and A_3 giving the partial fraction expansion

$$\frac{12}{s\,(s^2-8s+12)} \quad = \quad \frac{A_1}{s}+\frac{A_2}{s-2}+\frac{A_3}{s-6}.$$

   Find A_1, A_2 and A_3.

   (b) Find the inverse Laplace Transform (LT) of the partial fraction expansion in (a) and thus confirm the above.

2. Find the inverse LTs of the following functions:

   (a) $\dfrac{7}{s^4}$,

   (b) $\dfrac{1}{s}-\dfrac{2}{(s+3)^2-9}$,

   (c) $\dfrac{s-5}{s^2+4s+20}$,

   (d) $\dfrac{s^2-1}{s(s+2)(s-3)}$.

3. Suppose a function $y(t)$ satisfies the differential equation

$$-2\frac{\mathrm{d}y}{\mathrm{d}t}+10y \quad = \quad 4e^{-t}+2$$

   and the initial condition $y(0)=0$.

   (a) Use LTs to find $y(t)$.

   (b) Confirm that your answer to (a) satisfies both the differential equation and the initial condition.

Table of Laplace Transforms

$f(t) = L^{-1}[F(s)]$	$F(s) = L[f(t)]$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$

Laplace Transform Theorems

$$\begin{aligned} L\left[af(t)+bg(t)\right] &= aL\left[f(t)\right]+bL\left[g(t)\right], \\ L\left[e^{at}f(t)\right] &= F(s-a), \\ L\left[f(at)\right] &= \frac{1}{a}F\left(\frac{s}{a}\right), \\ L\left[f'(t)\right] &= sF(s)-f(0), \\ L\left[f''(t)\right] &= s^2F(s)-sf(0)-f'(0), \\ L\left[\int_0^t f(\tau)\,\mathrm{d}\tau\right] &= \frac{1}{s}F(s). \end{aligned}$$

In all of the above, $n = 0, 1, 2, \ldots$ and ω, a and b are constants.