EE112 – Engineering Mathematics II

Problem Set 12

Please note that this Problem Set is NOT required; you do not have to turn it in. I've made it available so that you have some problems to attempt that cover the material presented in the final week of lectures. However, since this material IS examinable, it's very definitely to your benefit to at least attempt to do these problems. Solutions will be put up on the website on Thursday 10 May.

1. Find the eigenvalues and eigenvectors of the following matrices:

$$\text{(a)} \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}, \qquad \text{(b)} \begin{pmatrix} 2 & 0 & -2 \\ 0 & 0 & -2 \\ -2 & -2 & 1 \end{pmatrix}.$$

   (Hint for (b): one of the eigenvalues is 1.)

2. (a) Compute the characteristic polynomials of the following matrices:

$$\text{(a)} \begin{pmatrix} 5 & -2 \\ 9 & -6 \end{pmatrix}, \qquad \text{(b)} \begin{pmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{pmatrix}.$$

   (b) Confirm the Cayley-Hamilton theorem for each of the matrices in (a), i.e. show that each satisfies its own characteristic equation.

3. Let A be the general 2×2 matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

   Using A's characteristic polynomial and the Cayley-Hamilton theorem, show that

$$A^2 = \begin{pmatrix} a^2 + bc & (a+d)b \\ (a+d)c & d^2 + bc \end{pmatrix}, \qquad A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

   Note: do *not* use any other techniques to find A^2 and A^{-1}, e.g. don't compute A^2 by multiplying A by itself or obtain A^{-1} by Gauss-Jordan reduction.