EE112 – Engineering Mathematics II

Problem Set 1

Due by 5pm on Friday, 9 February 2018

1. Show that the Laplace transforms (LTs) of $\cosh(at)$ and $\sin(\omega t)$, where a and ω are positive constants, are

$$L\left[\cosh(at)\right] = \frac{s}{s^2 - a^2}, \qquad L\left[\sin(\omega t)\right] = \frac{\omega}{s^2 + \omega^2}.$$

2. Find the LT of the function $f(t)$ given by

$$f(t) \quad = \quad \begin{cases} -2 & \text{for } 0 \leq t \leq 1, \\ 0 & \text{for } t > 1. \end{cases}$$

3. Find the LTs of the following functions:

   (a) $-t^3 + 2\sin(4t) + 2\cosh(2t)$;

   (b) $e^{-t}\cos(t) + 2t$;

   (c) $(2t - 1)^2$;

   (d) $t\cosh(2t)$;

   (e) $\sin^2(t)$;

   (f) $\sin(at + b)$, where a and b are constants.

   (Hint for (d): remember that $\cosh(x) = (e^x + e^{-x})/2$.)

Table of Laplace Transforms

$f(t)$	$F(s) = L[f(t)]$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$

Laplace Transform Theorems

$$\begin{aligned} L\left[af(t) + bg(t)\right] &= aL\left[f(t)\right] + bL\left[g(t)\right], \\ L\left[e^{at}f(t)\right] &= F(s-a), \\ L\left[f(at)\right] &= \frac{1}{a}F\left(\frac{s}{a}\right), \\ L\left[f'(t)\right] &= sF(s) - f(0), \\ L\left[f''(t)\right] &= s^2F(s) - sf(0) - f'(0), \\ L\left[\int_0^t f(\tau)\,\mathrm{d}\tau\right] &= \frac{1}{s}F(s). \end{aligned}$$

In all of the above, $n = 0, 1, 2, \ldots$ and ω, a and b are constants.