

Engineering mathematics 1 EE106

Problem Set 4

**** 1.** State Taylor's theorem for the expansion of a function

(i) About the point $x = 0$.

(ii) About the point $x = a$.

**** 2.** Use Taylor's theorem to show that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

**** 3.** Use the substitution $x = a \tan(\theta)$ to do the following integral

$$\int \frac{dx}{a^2 + x^2}$$

**** 4.** Do the integral

$$\int \frac{dx}{x \ln^2(x)}$$

*** 5.** State the formula for integration by parts.

***** 6.** Use integration by parts (twice) to evaluate

$$\int x^2 \sin(x) dx$$

**** 7.** Evaluate

$$\int_0^1 \ln(x) dx$$

* **8.** Write down the formula for the *volume* of a volume of revolution generated by the function $f(x)$ on an interval $[a, b]$.

** **9.** The ellipse

$$\frac{x^2}{4^2} + \frac{y^2}{6^2} = 1$$

has its upper half rotated about the x -axis so as to form an ellipsoid. Calculate the volume of this ellipsoid.

* **10.** Write down the formula for the *surface area* of a surface of revolution generated by the function $f(x)$ on an interval $[a, b]$.

*** **11.** The curve

$$f(x) = \cosh(x), \quad (0 \leq x \leq a)$$

is rotated about the x -axis to form a surface of revolution. Calculate the surface area of this surface between $x = 0$ and $x = a$. Look up any integral you need and remember $\cosh^2(x) - \sinh^2(x) = 1$.

** **12.** Write down the formula for the *length* of a curve $f(x)$ which starts at a and finishes at b .

Now calculate the *length* of the curve $f(x) = \cosh(x)$ between $x = 0$ and $x = a$.

* **13.** Write down the formula for the *mean* value of a function $f(x)$ on an interval $[a, b]$.

*** **14.** A radioactive element has a half life of 33,150 years. A sample of this element contains N atoms and N satisfies the differential equation.

$$\frac{dN}{dt} = -kN$$

Show that

$$N(t) = N_0 e^{-kt}$$

Find also the age of a sample for which 33% of the atoms have decayed.