EE106 Assignment – Engineering & Maths (5%)

Part 1 - Complex Numbers

The stability of a system is an important aspect of Engineering and a topic that will be touched upon in various modules throughout the Engineering Programme, including EE407 Control System Design. One aspect of this module looks at how stability can be measured for a given system, where the system is typically written in complex number form.

There are two stability measurements, namely Gain Margin (GM) and Phase Margin (PM). These are given by the following equations:

$$\text{GM} = \frac{1}{|\text{X}(\omega)|} \quad \text{when} \quad \arg(\text{X}(\omega)) = -180°$$

and

$$\text{PM} = 180° + \arg(\text{X}(\omega)) \quad \text{when} \quad |\text{X}(\omega)| = 1$$

Here, X represents the system in complex form and ω is a variable.

You will derive and understand these equations in the EE407 module. You do not need to understand them for this module.

Using this information, answer the following questions. Note that in the following questions, j is used instead of i to represent $\sqrt{-1}$. While i is correct, engineers tend to favour j as i typically represents current.

Q1 Consider the system given by: $\text{X}(\omega) = \dfrac{4}{j\omega(2 + j\omega)^2}$.

(a) Convert this system to polar format.

(b) Determine the Gain Margin of this system.

(c) Show that when $\omega = 0.848$ the magnitude of X (i.e. | X |) = 1.

(d) Hence determine the Phase Margin of the above system.

Q2 Consider the system given by: $\text{X}(\omega) = \dfrac{27}{(3 + j\omega)^3}$.

(a) Convert this system to polar format.

(b) Determine the Gain Margin of this system.

(c) Determine the Phase Margin of the above system.

Part 2 – Maxima / Maxima

Another important aspect of Engineering is optimisation. This is yet another topic that will be touched upon in various modules throughout the Engineering Programme and, in particular, EE403 Unconstrained Optimisation. Optimisation is important in determining the best solution to a given problem, for example – what is the cheapest solution, the highest point, the nearest exit, the fastest car, the most efficient system, etc?

In EE403, you will examine more complex and detailed methods for determining the optimal value of a given function. For now, it is sufficient to use differential calculus to determine the optimal points for simple functions.

Q3 The signalling range, y, of a submarine cable is given by:

$$y = kx^2 \ln\left(\frac{1}{x}\right)$$

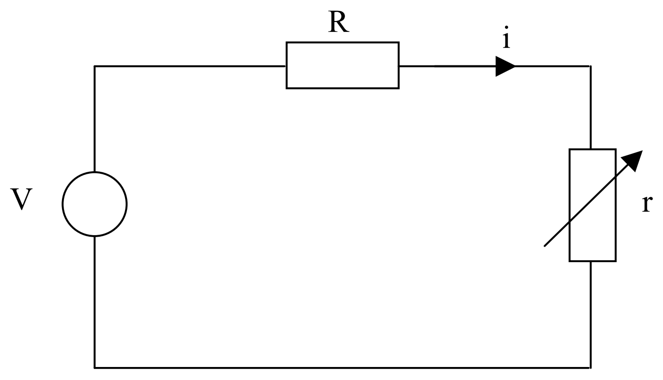
Here, x is the ratio of the radii of the conductor and cable and k is a positive constant value. Find the value of x for maximum range.

Q4 The power transmitted by a belt drive is:

$$P = k\left(Tv - \frac{wv^3}{10}\right)$$

Here, v is the belt speed, T is the tension on the driving side, w is the weight per unit length of belt and k is a positive constant value. Find the speed that maximises P.

Q5 Consider the following circuit, where the voltage V and resistance R are constant.



Given that the power dissipated in r is $P = i^2 r$ and that the current is $i = \frac{V}{r + R}$, find the maximum or minimum of P as r varies.