**BE in Electronic Engineering with Communications**
**BE in Electronic Engineering with Computers**
**BE in Electronic Engineering**
**BE in Robotics and Intelligent Devices**

# Year 1

# SEMESTER 1

# 2018–2019

# EE106

# Engineering Mathematics I

## Dr. P. Watts

Time allowed: 2 hours

Answer Question 1 and any two others

Question 1 carries 50 marks and all others carry 25 marks each

1. **This Question Is Compulsory**

   (a) Use the ratio test to show that the geometric series

   $$1 + \frac{x^2}{3} + \frac{x^4}{9} + \frac{x^6}{27} + \frac{x^8}{81} + \dots$$

   converges when $|x| < \sqrt{3}$.

   **(5 marks)**

   (b) Evaluate the following limit:

   $$\lim_{x \to \pi} \left[ \frac{x^2 - 2x}{2 - \cos(x)} \right].$$

   **(5 marks)**

   (c) Give the definition of the derivative of a function $f(x)$, and use this definition to differentiate the function

   $$f(x) \;=\; -x^2 + 10x + 3.$$

   **(10 marks)**

   (d) Find the critical points of the function

   $$f(x) \;=\; x^2(4 - x^2)$$

   and classify each of them as a maximum, a minimum or indeterminate.

   **(10 marks)**

   (e) Write down a solution to the differential equation

   $$\frac{\mathrm{d}y}{\mathrm{d}x} \;=\; 9y.$$

   **(5 marks)**

   (f) Calculate the area under the curve

   $$f(x) \;=\; x^2 - \frac{8}{x^3}$$

   from $x = 1$ to $x = 3$.

   **(5 marks)**

   (g) Evaluate the integral

   $$\int \frac{2x}{x^2 + 36} \, \mathrm{d}x.$$

   **(10 marks)**

2. (a) State l'Hôpital's rule and use it to evaluate the limit

$$\lim_{x \to 0} \left[ \frac{x}{\sinh(x)\cosh(3x)} \right].$$

**(10 marks)**

(b) State Taylor's theorem and use it to show that the first four terms in the Taylor expansion of $1/\sqrt{1-2x}$ around 0 are

$$\frac{1}{\sqrt{1-2x}} = 1 + x + \frac{3x^2}{2} + \frac{5x^3}{2} + \dots$$

**(15 marks)**

3. (a) Write down two independent solutions to the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 144y = 0.$$

**(10 marks)**

(b) Technetium-96 decays radioactively such that the number $N(t)$ of technetium-96 atoms in a sample at time $t$ obeys the differential equation

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = -kN(t)$$

where $k = 58.8\,\mathrm{y}^{-1}$. If $N_0$ is the initial number of atoms, solve this differential equation and use it to calculate the half-life (expressed in days) of Technetium-96.

**(15 marks)**

4. (a) The function

$$f(x) = 4\sqrt{\frac{x}{x^2+1}}$$

is rotated about the interval $[0, 1]$ to form a volume of revolution. Calculate its volume.

**(10 marks)**

(b) State the formula for integration by parts and use it to evaluate the integral

$$\int x \sinh(2x)\,\mathrm{d}x$$

**(15 marks)**