EE106 – Engineering Mathematics I

Problem Set 7

Due by 5pm on Friday, 23 November 2018

1. Suppose that $f(x)$ satisfies the differential equation (DE)

$$-f''(x) + 6f'(x) + 16f(x) = 0.$$

   (a) Find this DE's auxilary equation and use it to show that

   $$f(x) = Ae^{-2x} + Be^{8x}$$

   is a solution to the DE for any choice of the constants A and B.

   (b) Find the values of A and B such that $f(0) = -1$ and $f'(0) = 1$.

2. Write down two independent solutions to the DE

$$\frac{d^2y(t)}{dt^2} + 625y(t) = 0.$$

3. If α is a positive constant, confirm that

$$y(x) = \frac{3(\alpha - 1)\sin(x) - 3\cos(x)}{\alpha^2 - 2\alpha + 2}$$

   is a solution to

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + \alpha y = 3\sin(x)$$

   by computing the left-hand side and showing that it is indeed equal to $3\sin(x)$.

4. Consider the function

$$f(x) = h - \frac{hx}{b}.$$

   (a) Let $g(x)$ be the function

   $$g(x) = hx - \frac{hx^2}{2b}.$$

   Show that $g'(x) = f(x)$.

   (b) Use the Fundamental Theorem of Calculus to compute

   $$\int_0^b f(x)\,dx.$$

   (c) Make a plot of $f(x)$ between $x = 0$ and $x = b$; given this plot, and what quantity a definite integral represents, explain why you could have guessed the value of the integral in (b) based only on stuff you learned in secondary (or maybe even primary) school.

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