EE106 – Engineering Mathematics I

Problem Set 6

Due by 5pm on Friday, 16 November 2018

1. The curve given in plane polar coordinates by

$$r \quad = \quad \csc\left(\theta + \frac{\pi}{4}\right)$$

describes a straight line. Prove this by showing that in Cartesian coordinates, the above is equivalent to $y = ax + b$ for some constants a and b.

2. Let z be the complex number $1 + i$.

   (a) Compute z^*, z^2, $1/z$ and e^z, all expressed in Cartesian form, i.e. in the form $a + bi$.

   (b) Now express them all in polar form, i.e. in the form $re^{i\theta}$.

3. A radioactive element has a half-life of 127 days. The number of atoms in a sample of this element is given at time t by a function $N(t)$ that satisfies the differential equation (DE)

$$\frac{\mathrm{d}N}{\mathrm{d}t} \quad = \quad -kN$$

where k is the decay constant of the element.

   (a) Determine the value of k.

   (b) If the initial number of atoms in the sample is N_0, find the time it takes for one-third of the atoms to decay.

4. Consider the DE

$$\frac{\mathrm{d}y}{\mathrm{d}x} \quad = \quad y + \frac{1}{y}.$$

We wish to find the solution that satisfies $y(0) = 1$.

   (a) First, we define a new function $f(x) = [y(x)]^2 + 1$. Compute $f'(x)$ in terms of $y'(x)$.

   (b) Using the above DE for y, show that we now have a DE for f of the form

$$\frac{\mathrm{d}f}{\mathrm{d}x} \quad = \quad 2f.$$

   (c) Determine $f(0)$ and use it and the DE in (b) to find $f(x)$.

   (d) Remembering that $y(0) = 1$, find $y(x)$.