EE106 – Engineering Mathematics I

Problem Set 5

Due by 5pm on Friday, 9 November 2018

1. Use l'Hôpital's rule to evaluate the following limits:

$$\text{(a)} \qquad \lim_{x\to\infty} \frac{x^3+2x-7}{3x^3+x^2-x-1}$$

$$\text{(b)} \qquad \lim_{x\to 1} \frac{x-\frac{1}{x}}{x^2-\frac{2}{x+1}}$$

$$\text{(c)} \qquad \lim_{x\to 0} \frac{1-\sqrt{1+9x}}{\sqrt{x}}$$

2. Show that the first three nonzero terms in the Taylor series expansion of $1/(1-2x^2)$ around the point $a=0$ are

$$\frac{1}{1-2x^2} \quad = \quad 1+2x^2+4x^4+\dots$$

3. Show that the first three nonzero terms in the Taylor series expansion of $\sin(x)$ around the point $a=\pi/2$ are

$$\sin(x) \quad = \quad 1-\frac{1}{2}\left(x-\frac{\pi}{2}\right)^2+\frac{1}{24}\left(x-\frac{\pi}{2}\right)^4+\dots$$

4. The Taylor series expansion of $\ln(x)$ around the point $a=1$ is

$$\ln(x) \quad = \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}.$$

(You don't have to show this.) Use the first four terms of this series to obtain an approximate value for $\ln(e^{-2})$ (where $e \approx 2.7182818$). How close is this – expressed as a percentage – to the actual value of -2?