**Problem Set 2**

Due by 5pm on Friday, 12 October 2018

1. (a) The total resistance of $N$ resistors $R_1, R_2, \ldots, R_N$ connected in series is

$$
\begin{aligned}
R_{\text{total}} &= R_1 + R_2 + \ldots + R_N \\
&= \sum_{i=1}^{N} R_i.
\end{aligned}
$$

Suppose the $i^{\text{th}}$ resistance is given by the formula

$$
R_i = \frac{2^i Z}{3^i}
$$

where $Z = 2$ ohms; what is the total resistance of an *infinite* number of these resistors connected in series?

(b) Construct an infinite geometric series whose first term is 3 and whose sum is 27. In other words, give appropriate values for $a$ and $r$.

2. (a) Use the comparison test to prove that the series

$$
\sum_{n=1}^{\infty} \frac{1}{2^n + 3n}
$$

converges. (Hint: think of a convergent geometric series that you can compare it to.)

(b) Use the ratio test to show that the series

$$
\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}
$$

converges if $|x|$ is less than 1.

3. For each of the following functions, state if the given limit exists, and if it does, find it:

(a) $\quad \lim\limits_{x \to -1} \left( x^2 - 13x + 25 \right)$

(b) $\quad \lim\limits_{x \to 2} \dfrac{2x}{3x - 7}$

(c) $\quad \lim\limits_{x \to -\infty} \dfrac{1}{1 - e^{2x}}$

4. (a) Plot the function $f(x) = |x| - 1$. Is it continuous or discontinuous at $x = 0$?

(b) Sketch a graph which depicts a function $g(x)$ which has discontinuities at 0, 1.5 and $-1$ but is continuous at all other values of $x$.