Solutions to EE106 Problem Set #1

P.1

The key to answering this question is to look at both the difference and the ratio of successive terms in each series: if the difference is always the same, it's an arithmetic series. If the ratio is always the same, it's geometric. And if neither of these is true, it's neither arithmetic nor geometric. So let's begin:

(a) Difference: $4023-4000=23$

$4046-4023=23$

$4069-4046=23$

$\vdots$

So it looks arithmetic. To make sure, we have to confirm that the last term, 4230, is the first term plus an integer multiple of 23. And it is:

$4230-4023=207=9\times 23$

So we have confirmed that this series is $\boxed{\text{arithmetic}}$.

(Note: once we've determined a series is arithmetic, we don't have to check to see if it's geometric, or vice versa. This is because the only series which are both arithmetic and geometric is

$a+a+a+a+a+a+ \ldots +a$

which has a difference $d=0$ and a ratio $r=1$. No other series is both.)

(b) Difference: $0.12-0.1=0.02$

$0.123-0.12=0.003$

... and we can stop there. Since these differences are not the same, we can already say it's not arithmetic.

Ratio: $0.12/0.1=1.2$

$0.123/0.12=1.025$

... and, again, we can stop. The ratio is different, so it can't be geometric either. Thus, this series is $\boxed{\text{neither arithmetic nor geometric}}$

(c) Difference: $12.3 - 1.23 = 11.07$

$$1.23 - 0.123 = 1.107$$

which is obviously different. So this isn't arithmetic. However, the ratios are:

$$12.3/1.23 = 10$$

$$1.23/0.123 = 10$$

$$0.123/0.0123 = 10$$

... so it does have a constant ratio between successive terms. Thus, this series is $\boxed{\text{geometric}}$.

P.2

You are completely free to pick any a and any r you like for this problem. Me, I'll take $a = 0.1$ and $r = -2$ for no good reason. Thus, a six-term geometric series that starts with $a = 0.1$ and has a ratio of $r = -2$ is

$$\boxed{0.1 - 0.2 + 0.4 - 0.8 + 1.6 - 3.2}$$

I could, of course, add this by hand, but let's use our formula instead: The sum should be

$$a\frac{1-r^n}{1-r} = (0.1)\,\frac{1-(-2)^6}{1-(-2)} = 0.1 \cdot \left(\frac{-63}{3}\right) = 0.1 \cdot (-21) = \boxed{-2.1}$$

which, in fact, it is.

P.3

Again, I can make up any a and r I like, but remember that $|r| < 1$ if it is going to be convergent. So I'm going to pick a particularly weird one where $a = 42$ and $r = \frac{1}{23}$, namely,

$$\boxed{42 + \frac{42}{23} + \frac{42}{529} + \frac{42}{12167} + \frac{42}{279841} + \cdots}$$

Now, to find its sum, we use the formula $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$, i.e.

$$\sum_{n=1}^{\infty} \frac{42}{23^{n-1}} = \frac{42}{1-1/23} = \frac{42}{22/23} = \boxed{\frac{483}{11}}$$

or 43.90909... if you prefer decimals. (However, 483/11 is exact, whereas 43.90909... isn't, so the fraction is actually better.)

P. 4

This is yet another infinite geometric series, so we get to use the formula from P. 3 again. But we have to find a and r first. However, both are easy: we're told the total resistance will be

$$R_{total} = R_1 + R_2 + R_3 + \dots$$
$$= \sum_{i=1}^{\infty} R_i$$

where $R_i = 3\Omega/5^i$. So the first term, R_1, is $a = R_1 = \frac{3\Omega}{5} = 0.6\Omega$.

The ratio between successive resistors is

$$R_{i+1}/R_i = \frac{3\Omega/5^{i+1}}{3\Omega/5^i} = \frac{1}{5}$$

so $r = 1/5$. We have all the data we need, and we plug them into the formula:

$$R_{total} = \frac{a}{1-r} = \frac{0.6\Omega}{1-1/5} = \frac{0.6\Omega}{4/5} = \boxed{0.75\Omega.}$$