



OLLSCOIL NA hÉIREANN MÁ NUAD
THE NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

BE in Electronic Engineering with Communications
BE in Electronic Engineering with Computers
BE in Electronic Engineering
BA in Finance & Venture Management

Year 1

Semester 1

2014 - 2015

Engineering Mathematics I
EE106

Dr. Paul Watts

Time allowed: 2 hours

Answer Question 1 and any two others

Question 1 carries 50 marks and all others carry 25 marks each

1. **This Question Is Compulsory**

- (a) [5 marks] Use the ratio test to show that the series

$$1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \dots$$

converges when $|x| > 1$.

- (b) [5 marks] Evaluate the following limit:

$$\lim_{x \rightarrow -2} \left(\frac{x-2}{x^2-1} \tan(x) \right)$$

- (c) [5 marks] Give the definition of the derivative of a function $f(x)$, and use this definition to differentiate the function

$$f(x) = 2(x-5)^2$$

- (d) [10 marks] Find the critical points of the function

$$f(x) = x(x-3)^3$$

and classify each of them as a maximum, a minimum or neither.

- (e) [5 marks] Write down a solution to the differential equation

$$\frac{dy}{dx} = 10y$$

- (f) [10 marks] Calculate the area under the curve

$$f(x) = \frac{2}{x} - \frac{1}{x^2}$$

from $x = 1$ to $x = 10$.

- (g) [10 marks] Use a trigonometric substitution to evaluate the integral

$$\int \frac{dx}{4+x^2}$$

2. (a) [10 marks] State l'Hôpital's rule and use it to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{(\sinh(3x))^2}{x^2}$$

- (b) [15 marks] State Taylor's theorem and use it to show that the first four terms in the Taylor expansion of $\sqrt{1+x}$ around 0 are

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \dots$$

3. (a) [10 marks] Write down two independent solutions to the differential equation

$$\frac{d^2y}{dx^2} + 25y = 0$$

- (b) [15 marks] Plutonium-244 decays with time and the number $N(t)$ of atoms in a sample at time t obeys the differential equation

$$\frac{dN(t)}{dt} = -kN(t)$$

where $k = 8.66 \times 10^{-9} \text{ y}^{-1}$. Solve this differential equation and use it to calculate the half-life of plutonium-244.

4. (a) [10 marks] The function

$$f(x) = x(1-x)$$

is rotated about the interval $[0, 1]$ to form a volume of revolution. Calculate its volume.

- (b) [15 marks] State the formula for integration by parts and use it to evaluate the integral

$$\int x \sin(x) dx$$