EE106 – Engineering Mathematics I

Problem Set 5

Due in tutorial on Thursday, 13 November 2014

1. If $f(x)$ and $g(x)$ are the two functions

$$f(x) = \exp\left\{(\cos(x))^2\right\}, \qquad g(x) = \exp\left\{(\sin(x))^2\right\},$$

   then compute the derivative of $f(x)g(x)$. Explain why it is unnecessary, although perfectly permissible, to use the product rule in your computation.

2. Show that the first three nonzero terms in the Taylor series expansion of $1/(1+x^2)$ around the point $a = 0$ are

$$\frac{1}{1+x^2} \quad = \quad = 1 - x^2 + x^4 + \dots$$

3. The Taylor series expansion of $\ln(x)$ around the point $a = 1$ is

$$\ln(x) \quad = \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}.$$

   Use the first three terms of this series to obtain an approximate value for $\ln(e^{-1})$ (where $e \approx 2.7182818$). How close is this – expressed as a percentage – to the actual value of -1?

4. Let z be the complex number $2+3i$. Compute z^*, z^2 and e^z, all expressed in the form $a + bi$.
   (Hint for the third computation: remember that $e^{x+yi} = e^x e^{iy}$.)