EE106 – Engineering Mathematics I

Problem Set 4

Due in tutorial on Thursday, 6 November 2014

1. If $f(x) = \arctan(x)$, then f satisfies

$$\tan\left(f(x)\right) \quad = \quad x.$$

Use this to show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\arctan(x) \quad = \quad \frac{1}{1+x^2}$$

Remember that $\tan(x) = \sin(x)/\cos(x)$ and that $(\cos(x))^2 + (\sin(x))^2 = 1$.

2. Find all the critical points of the following functions, and identify them as maxima, minima or neither. Then determine if they have any points of inflection and, if so, where they occur.

(a) $f(x) = x^3 - 18x^2 - 39x + 100$

(b) $f(x) = \dfrac{x}{x^2+1}$

(c) $f(x) = x^6$

(d) $f(x) = e^{-x^2}$

3. An alternating voltage is described by the function

$$V(t) \quad = \quad V_0\left[\cos(\omega t)\right]^2$$

where V_0 and ω are constants. Determine the maximum and minimum values of this voltage.

4. Use l'Hôpital's rule to evaluate the following limits:

(a) $\displaystyle\lim_{x\to\infty} \frac{5x^3 + 4x - 7}{7x^3 + 5x^2 - x + 1}$

(b) $\displaystyle\lim_{x\to 0}\left(\frac{\frac{x}{7} + \frac{11}{x^3}}{\frac{x^2}{2} - \frac{2}{x^3}}\right)$

(c) $\displaystyle\lim_{x\to\infty}\left[(x+27)^{1/3} - x^{1/3}\right]$

(**Hint for (c):** Change the variable from x to $y = 1/x$, rearrange into a quotient of two functions and then use l'Hôpital's rule.)