EE106 – Engineering Mathematics I

Problem Set 10

Due in tutorial on Thursday, 18 December 2014

1. Find the mean value of the function

$$f(x) \quad = \quad x^5 - \frac{2}{x^3}$$

over the interval $[1, 2]$.

2. A decaying voltage has the form

$$V(t) \quad = \quad V_0 e^{-t} \sin(2t)$$

where V_0 is a positive constant. Find the root-mean-squared (RMS) value of this voltage between $t = 0$ and $t = \pi$. Remember that

$$(\sin(\theta))^2 \quad = \quad \frac{1}{2} - \frac{1}{2}\cos(2\theta)$$

for any angle θ.

3. Use Newton's method to obtain an estimate for the zero of the cubic function

$$f(x) \quad = \quad 2x^3 - x^2 + 10x - 5$$

by starting with an initial guess of $x_1 = 1$. Stop when you obtain two successive estimates that differ by less than 0.001. (You should get to this level of precision only after a few iterations.)

4. Simpson's rule says that if we choose to divide the interval $[a, b]$ into four regions, then

$$\int_a^b f(x)\,\mathrm{d}x \quad \approx \quad \frac{\Delta x}{3}\left[f\left(x_1\right) + 4f\left(x_2\right) + 2f\left(x_3\right) + 4f\left(x_4\right) + f\left(x_5\right)\right]$$

where $x_1 = a$, $x_5 = b$ and

$$x_i = a + (i-1)\Delta x, \qquad \Delta x = \frac{b-a}{4}$$

Use this to estimate the integral

$$\int_1^2 \frac{1}{x}\,\mathrm{d}x$$

and compare your result with the exact answer of $\ln(2)$.